# Non-Parametric Statistics for Research in Conservation

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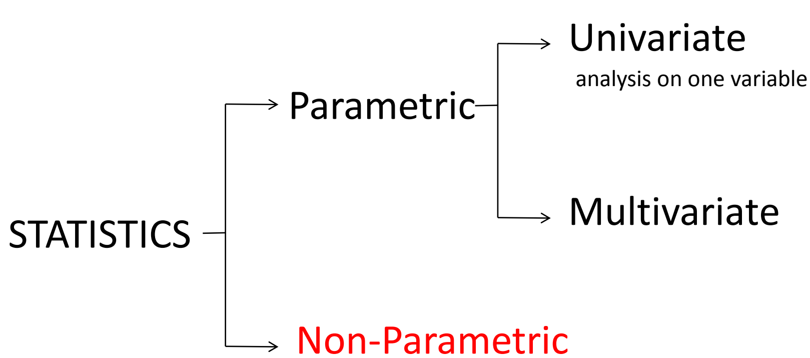
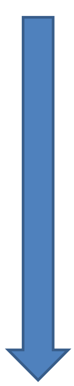


## When you need to use non-parametric statistics?

DEPEND ON:

- Scales of measurements
- Distribution of your data
  - parametric stats: data distributions are approximately normal
  - need to check the skew and kurtosis
  - for a relatively normal distribution: skew ~= 1.0, kurtosis~=1.0
- Number of samples: small samples → difficult to assume normality

## Scales of Measurements



- Nominal: naming
- Ordinal: ordering
- Interval: equal interval without absolute zero
  - temperature, calendar year
- Ratio: equal interval with absolute zero

higher

| | Nominal | Ordinal | Interval | Ratio |
|---|---|---|---|---|
| *Sex* | x | | | |
| *Hair colour* | x | | | |
| *Pulse* | | | | x |
| *Temp. °C* | | | x | |
| *Team number* | x | | | |
| *Shoe size* | | x | | |
| *Year of birth* | | | x | |
| *Friends* | x | | | |
| *Fruit/veg* | x | | | |
| *Health* | | x | | |
| *Pulse* | | | | x |
| *Temp.* | | | x | |
| *Residence* | x | | | |

# When you need to use non-parametric statistics?

- If your measurement is interval or ratio scales → use parametric statistics

  → *Parametric stats cannot be used on rank data!*
- If you are using nominal or ordinal →use non-parametric statistics
- Non-parametric statistics: use less information in calculation →less powerful

  → Data on ratio scales can be analyzed by using non-parametric stats

| Type of Statistic | | Scale of Measurement |
|---|---|---|
| Nonparametric | - | Nominal |
| Nonparametric | - | Ordinal |
| Nonparametric | Parametric | Approximately Interval |
| - | Parametric | Interval |
| - | Parametric | Ratio |

# Non-Parametric Statistics

- "Distribution-free statistics"
- No assumptions about the distribution of the data (e.g., normality)
- Use RANKS rather than their raw values → we might loose some important info

"How much do we want to loose information by transforming numbers into ranks?"

# Non-Parametric Statistics

- Calculation: simple, fast and easy
- No need to check or assume normality
- Good for small samples

BUT:

- Tests are not as varied as parametric stats, some tests (e.g., regression/prediction) are unavailable
- Again .. Less powerful than parametric stats

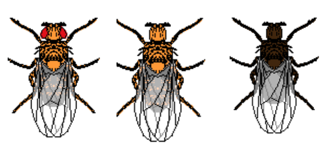
| Nominal Data | Ordinal Data |
|---|---|
| Chi-Square Goodness-of-Fit Test | Mann-Whitney *U* Test |
| Chi-Square Test of Independence | Wilcoxon *T* Test |
| McNemar Test | Kruskal-Wallis *H* Test |
| | Friedman ANOVA by Ranks |
| | Spearman's $r_s$ |

# Chi-Square

- To compare observed data with data we would expect to obtain according to a specific hypothesis
- Actually is a non-parametric statistics, although we also learn this test in parametric stats
- Chi-square → nominal data → frequency
- Formula:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$O_i \equiv$ observed values for each group
$E_i \equiv$ expected values for each group



# Heritability if Fruit flies

| Phenotype | Observed | Hypothesis | |
|---|---|---|---|
| Wild | 5610 | 5634 | 9 |
| Eyeless | 1881 | 1878 | 3 |
| Black body | 1896 | 1878 | 3 |
| Eyeless, black body | 622 | 626 | 1 |
| Total | 10009 | | |

# Mann–Whitney *U* test

- Also known as Wilcoxon Rank-Sum Test
- The *U* test is useful in the same situations as the independent samples Student's *t*-test
- One of the best-known non-parametric significance tests
- Formula: (select the highest; be careful in using critical value table)

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

Nominal Tail Probabilities: Directional $H_A$ $\alpha = 0.025$; Non-directional $H_A$ $\alpha = 0.05$

| $n_1 \backslash n_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | | | |
| 2 | | | | | | | | 16 | 18 | 20 | 22 | 23 | 25 | 27 | 29 | 31 |
| 3 | | | | | 15 | 17 | 20 | 22 | 25 | 27 | 30 | 32 | 35 | 37 | 40 | 42 |
| 4 | | | | 16 | 19 | 22 | 25 | 28 | 32 | 35 | 38 | 41 | 44 | 47 | 50 | 53 |
| 5 | | | 15 | 19 | 23 | 27 | 30 | 34 | 38 | 42 | 46 | 49 | 53 | 57 | 61 | 65 |
| 6 | | | 17 | 22 | 27 | 31 | 36 | 40 | 44 | 49 | 53 | 58 | 62 | 67 | 71 | 75 |
| 7 | | | 20 | 25 | 30 | 36 | 41 | 46 | 51 | 56 | 61 | 66 | 71 | 76 | 81 | 86 |
| 8 | | 16 | 22 | 28 | 34 | 40 | 46 | 51 | 57 | 63 | 69 | 74 | 80 | 86 | 91 | 97 |
| 9 | | 18 | 25 | 32 | 38 | 44 | 51 | 57 | 64 | 70 | 76 | 82 | 89 | 95 | 101 | 107 |
| 10 | | 20 | 27 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 97 | 104 | 111 | 118 |
| 11 | | 22 | 30 | 38 | 46 | 53 | 61 | 69 | 76 | 84 | 91 | 99 | 106 | 114 | 121 | 129 |
| 12 | | 23 | 32 | 41 | 49 | 58 | 66 | 74 | 82 | 91 | 99 | 107 | 115 | 123 | 131 | 139 |
| 13 | | 25 | 35 | 44 | 53 | 62 | 71 | 80 | 89 | 97 | 106 | 115 | 124 | 132 | 141 | 149 |
| 14 | | 27 | 37 | 47 | 57 | 67 | 76 | 86 | 95 | 104 | 114 | 123 | 132 | 141 | 151 | 160 |
| 15 | | 29 | 40 | 50 | 61 | 71 | 81 | 91 | 101 | 111 | 121 | 131 | 141 | 151 | 161 | 170 |
| 16 | | 31 | 42 | 53 | 65 | 75 | 86 | 97 | 107 | 118 | 129 | 139 | 149 | 160 | 170 | 181 |
| 17 | | 32 | 45 | 57 | 68 | 80 | 91 | 102 | 114 | 125 | 136 | 147 | 158 | 169 | 180 | 191 |
| 18 | | 34 | 47 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 143 | 155 | 167 | 178 | 190 | 202 |
| 19 | | 36 | 50 | 63 | 76 | 89 | 101 | 114 | 126 | 138 | 151 | 163 | 175 | 188 | 200 | 212 |
| 20 | | 38 | 52 | 66 | 80 | 93 | 106 | 119 | 132 | 145 | 158 | 171 | 184 | 197 | 210 | 222 |

# Wilcoxon *T* Test

- Also known as Wilcoxon paired-sample test
- To compare two related samples, matched samples, or repeated measurements on a single sample
- To assess whether their population mean ranks differ → paired difference test
- Alternative of paired sample t-test

Table W Critical Values of the Wilcoxon *T* Test

| | Level of Significance for a One-Tailed Test | | | | | Level of Significance for a One-Tailed Test | | | |
|---|---|---|---|---|---|---|---|---|---|
| | .05 | .025 | .01 | .005 | | .05 | .025 | .01 | .005 |
| | Level of Significance for a Two-Tailed Test | | | | | Level of Significance for a Two-Tailed Test | | | |
| *n* | .10 | .05 | .02 | .01 | *n* | .10 | .05 | .02 | .01 |
| 5 | 0 | — | — | — | 28 | 130 | 116 | 101 | 91 |
| 6 | 2 | 0 | — | — | 29 | 140 | 126 | 110 | 100 |
| 7 | 3 | 2 | 0 | — | 30 | 151 | 137 | 120 | 109 |
| 8 | 5 | 3 | 1 | 0 | 31 | 163 | 147 | 130 | 118 |
| 9 | 8 | 5 | 3 | 1 | 32 | 175 | 159 | 140 | 128 |
| 10 | 10 | 8 | 5 | 3 | 33 | 187 | 170 | 151 | 138 |
| 11 | 13 | 10 | 7 | 5 | 34 | 200 | 182 | 162 | 148 |
| 12 | 17 | 13 | 9 | 7 | 35 | 213 | 195 | 173 | 159 |
| 13 | 21 | 17 | 12 | 9 | 36 | 227 | 208 | 185 | 171 |
| 14 | 25 | 21 | 15 | 12 | 37 | 241 | 221 | 198 | 182 |
| 15 | 30 | 25 | 19 | 15 | 38 | 256 | 235 | 211 | 194 |
| 16 | 35 | 29 | 23 | 19 | 39 | 271 | 249 | 224 | 207 |
| 17 | 41 | 34 | 27 | 23 | 40 | 286 | 264 | 238 | 220 |
| 18 | 47 | 40 | 32 | 27 | 41 | 302 | 279 | 252 | 233 |
| 19 | 53 | 46 | 37 | 32 | 42 | 319 | 294 | 266 | 247 |
| 20 | 60 | 52 | 43 | 37 | 43 | 336 | 310 | 281 | 261 |
| 21 | 67 | 58 | 49 | 42 | 44 | 353 | 327 | 296 | 276 |
| 22 | 75 | 65 | 55 | 48 | 45 | 371 | 343 | 312 | 291 |
| 23 | 83 | 73 | 62 | 54 | 46 | 389 | 361 | 328 | 307 |
| 24 | 91 | 81 | 69 | 61 | 47 | 407 | 378 | 345 | 322 |
| 25 | 100 | 89 | 76 | 68 | 48 | 426 | 396 | 362 | 339 |
| 26 | 110 | 98 | 84 | 75 | 49 | 446 | 415 | 379 | 355 |
| 27 | 119 | 107 | 92 | 83 | 50 | 466 | 434 | 397 | 373 |

# Kruskal-Wallis

- Identical to a one-way ANOVA with the data replaced by their ranks
- to decide whether *k* independent samples are from different populations
- An extension of the Mann-Whitney *U* test to 3 or more groups
- Formula:

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1)$$

K = 3

| Sample Sizes | | | α = 0.05 | α = 0.01 |
|---|---|---|---|---|
| 2 | 2 | 2 | - | - |
| 3 | 2 | 1 | - | - |
| 3 | 2 | 2 | 4.714 | - |
| 3 | 3 | 1 | 5.143 | - |
| 3 | 3 | 2 | 5.361 | - |
| 3 | 3 | 3 | 5.600 | 7.200 |
| 4 | 2 | 1 | - | - |
| 4 | 2 | 2 | 5.333 | - |
| 4 | 3 | 1 | 5.208 | - |
| 4 | 3 | 2 | 5.444 | 6.444 |
| 4 | 3 | 3 | 5.791 | 6.745 |
| 4 | 4 | 1 | 4.967 | 6.667 |
| 4 | 4 | 2 | 5.455 | 7.036 |
| 4 | 4 | 3 | 5.598 | 7.144 |
| 4 | 4 | 4 | 5.692 | 7.654 |
| 5 | 2 | 1 | 5.000 | - |
| 5 | 2 | 2 | 5.160 | 6.533 |
| 5 | 3 | 1 | 4.960 | - |
| 5 | 3 | 2 | 5.251 | 6.909 |
| 5 | 3 | 3 | 5.648 | 7.079 |
| 5 | 4 | 1 | 4.985 | 6.955 |
| 5 | 4 | 2 | 5.273 | 7.205 |
| 5 | 4 | 3 | 5.656 | 7.445 |
| 5 | 4 | 4 | 5.657 | 7.760 |
| 5 | 5 | 1 | 5.127 | 7.309 |
| 5 | 5 | 2 | 5.338 | 7.338 |
| 5 | 5 | 3 | 5.705 | 7.578 |
| 5 | 5 | 4 | 5.666 | 7.823 |
| 5 | 5 | 5 | 5.780 | 8.000 |
| 6 | 1 | 1 | - | - |

| | | | | |
|---|---|---|---|---|
| 5 | 5 | 5 | 5.780 | 8.000 |

K = 4

| Sample sizes | | | | α = 0.05 | α = 0.01 |
|---|---|---|---|---|---|
| 2 | 2 | 1 | 1 | - | - |
| 2 | 2 | 2 | 1 | 5.679 | - |
| 2 | 2 | 2 | 2 | 6.167 | 6.667 |
| 3 | 1 | 1 | 1 | - | - |
| 3 | 2 | 1 | 1 | - | - |
| 3 | 2 | 2 | 1 | 5.833 | - |
| 3 | 2 | 2 | 2 | 6.333 | 7.133 |
| 3 | 3 | 1 | 1 | 6.333 | - |
| 3 | 3 | 2 | 1 | 6.244 | 7.200 |
| 3 | 3 | 2 | 2 | 6.527 | 7.636 |
| 3 | 3 | 3 | 1 | 6.600 | 7.400 |
| 3 | 3 | 3 | 2 | 6.727 | 8.015 |
| 3 | 3 | 3 | 3 | 7.000 | 8.538 |
| 4 | 1 | 1 | 1 | - | - |
| 4 | 2 | 1 | 1 | 5.833 | - |
| 4 | 2 | 2 | 1 | 6.133 | 7.000 |
| 4 | 2 | 2 | 2 | 6.545 | 7.391 |
| 4 | 3 | 1 | 1 | 6.178 | 7.067 |
| 4 | 3 | 2 | 1 | 6.309 | 7.455 |
| 4 | 3 | 2 | 2 | 6.621 | 7.871 |
| 4 | 3 | 3 | 1 | 6.545 | 7.758 |
| 4 | 3 | 3 | 2 | 6.795 | 8.333 |
| 4 | 3 | 3 | 3 | 6.984 | 8.659 |
| 4 | 4 | 1 | 1 | 5.945 | 7.909 |
| 4 | 4 | 2 | 1 | 6.386 | 7.909 |
| 4 | 4 | 2 | 2 | 6.731 | 8.346 |
| 4 | 4 | 3 | 1 | 6.635 | 8.231 |
| 4 | 4 | 3 | 2 | 6.874 | 8.621 |
| 4 | 4 | 3 | 3 | 7.038 | 8.876 |
| 4 | 4 | 4 | 1 | 6.725 | 8.588 |

K = 5

| Sample sizes | | | | | α = 0.05 | α = 0.01 |
|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 1 | 1 | - | - |
| 2 | 2 | 2 | 1 | 1 | 6.750 | - |
| 2 | 2 | 2 | 2 | 1 | 7.133 | 7.533 |
| 2 | 2 | 2 | 2 | 2 | 7.418 | 8.291 |
| 3 | 1 | 1 | 1 | 1 | - | - |
| 3 | 2 | 1 | 1 | 1 | 6.583 | - |
| 3 | 2 | 2 | 1 | 1 | 6.800 | 7.600 |
| 3 | 2 | 2 | 2 | 1 | 7.309 | 8.127 |
| 3 | 2 | 2 | 2 | 2 | 7.682 | 8.682 |
| 3 | 3 | 1 | 1 | 1 | 7.111 | - |
| 3 | 3 | 2 | 1 | 1 | 7.200 | 8.073 |
| 3 | 3 | 2 | 2 | 1 | 7.591 | 8.576 |
| 3 | 3 | 2 | 2 | 2 | 7.910 | 9.115 |
| 3 | 3 | 3 | 1 | 1 | 7.576 | 8.424 |
| 3 | 3 | 3 | 2 | 1 | 7.769 | 9.051 |
| 3 | 3 | 3 | 2 | 2 | 8.044 | 9.505 |
| 3 | 3 | 3 | 3 | 1 | 8.000 | 9.451 |
| 3 | 3 | 3 | 3 | 2 | 8.200 | 9.876 |
| 3 | 3 | 3 | 3 | 3 | 8.333 | 10.20 |

# Friedman Test

- To compare observations repeated on the same subjects
- A non-parametric randomized block ANOVA
- Formula:

$$M = \frac{12}{nk(k+1)} \sum R_j^2 - 3n(k+1)$$

**Critical values for the Friedman Test**

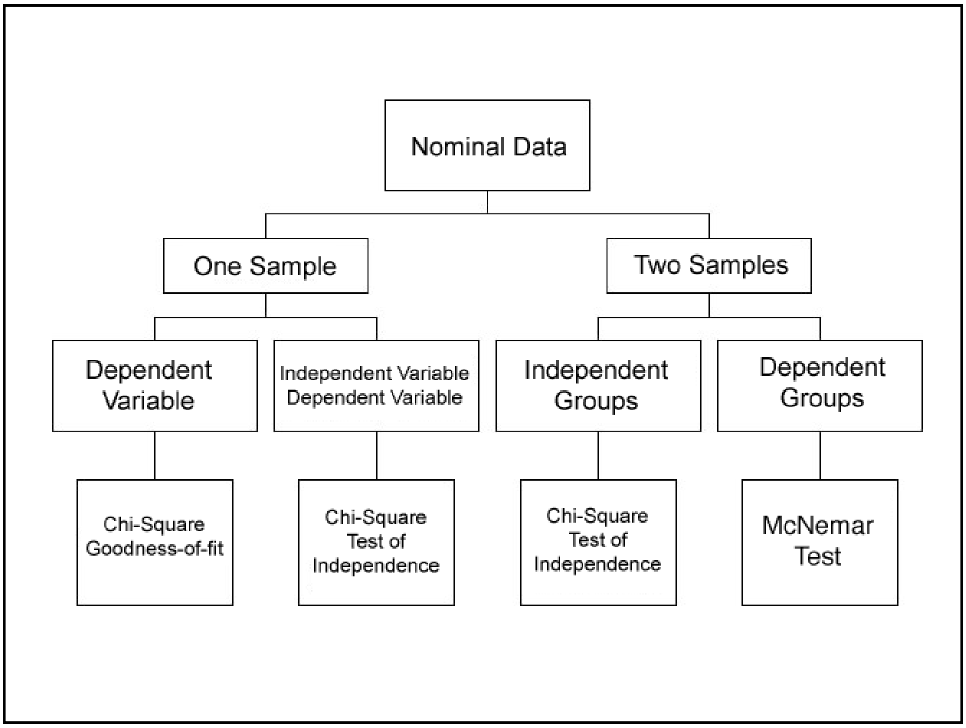
$$M = \frac{12}{nk(k+1)} \sum R_j^2 - 3n(k+1)$$

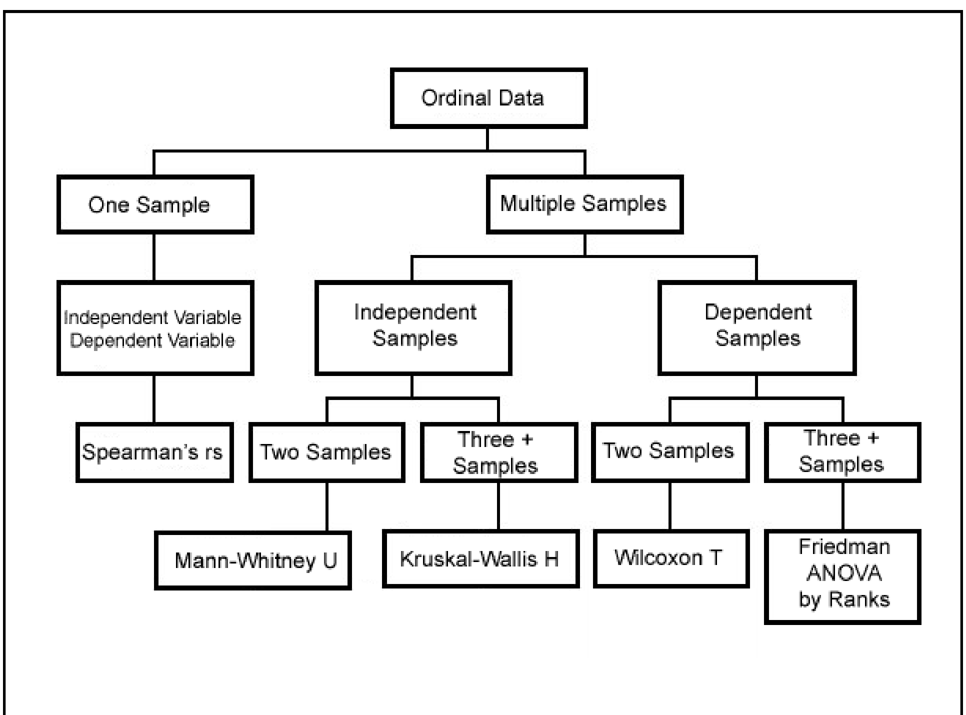
| | k=3 | | k=4 | | k=5 | | k=6 | |
|---|---|---|---|---|---|---|---|---|
| n | α=5% | α=1% | α=5% | α=1% | α=5% | α=1% | α=5% | α=1% |
| 2 | — | — | 6.000 | — | 7.600 | 8.000 | 9.143 | 9.714 |
| 3 | 6.000 | — | 7.400 | 9.000 | 8.533 | 10.130 | 9.857 | 11.760 |
| 4 | 6.500 | 8.000 | 7.800 | 9.600 | 8.800 | 11.200 | 10.290 | 12.710 |
| 5 | 6.400 | 8.400 | 7.800 | 9.960 | 8.960 | 11.680 | 10.490 | 13.230 |
| 6 | 7.000 | 9.000 | 7.600 | 10.200 | 9.067 | 11.870 | 10.570 | 13.620 |
| 7 | 7.143 | 8.857 | 7.800 | 10.540 | 9.143 | 12.110 | 10.670 | 13.860 |
| 8 | 6.250 | 9.000 | 7.650 | 10.500 | 9.200 | 13.200 | 10.710 | 14.000 |
| 9 | 6.222 | 9.556 | 7.667 | 10.730 | 9.244 | 12.440 | 10.780 | 14.140 |
| 10 | 6.200 | 9.600 | 7.680 | 10.680 | 9.280 | 12.480 | 10.800 | 14.230 |
| 11 | 6.545 | 9.455 | 7.691 | 10.750 | 9.309 | 12.580 | 10.840 | 14.320 |
| 12 | 6.500 | 9.500 | 7.700 | 10.800 | 9.333 | 12.600 | 10.860 | 14.380 |
| 13 | 6.615 | 9.385 | 7.800 | 10.850 | 9.354 | 12.680 | 10.890 | 14.450 |

# Spearman Rank Correlation Coefficient

- To test the direction and strength of the relationship between two variables
- Using ranks to calculate the correlation
- Identical to Pearson Product Moment
- Compute correlation only; unable to compute regression!
- Formula: $$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

| | Nominal $\alpha$ | | | | | |
|---|---|---|---|---|---|---|
| $n$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
| 4 | 1.000 | 1.000 | - | - | - | - |
| 5 | 0.800 | 0.900 | 1.000 | 1.000 | - | - |
| 6 | 0.657 | 0.829 | 0.886 | 0.943 | 1.000 | - |
| 7 | 0.571 | 0.714 | 0.786 | 0.893 | 0.929 | 1.000 |
| 8 | 0.524 | 0.643 | 0.738 | 0.833 | 0.881 | 0.952 |
| 9 | 0.483 | 0.600 | 0.700 | 0.783 | 0.833 | 0.917 |
| 10 | 0.455 | 0.564 | 0.648 | 0.745 | 0.794 | 0.879 |
| 11 | 0.427 | 0.536 | 0.618 | 0.709 | 0.755 | 0.845 |
| 12 | 0.406 | 0.503 | 0.587 | 0.678 | 0.727 | 0.818 |
| 13 | 0.385 | 0.484 | 0.560 | 0.648 | 0.703 | 0.791 |
| 14 | 0.367 | 0.464 | 0.538 | 0.626 | 0.679 | 0.771 |
| 15 | 0.354 | 0.446 | 0.521 | 0.604 | 0.654 | 0.750 |
| 16 | 0.341 | 0.429 | 0.503 | 0.582 | 0.635 | 0.729 |
| 17 | 0.328 | 0.414 | 0.488 | 0.566 | 0.618 | 0.711 |
| 18 | 0.317 | 0.401 | 0.472 | 0.550 | 0.600 | 0.692 |
| 19 | 0.309 | 0.391 | 0.460 | 0.535 | 0.584 | 0.675 |
| 20 | 0.299 | 0.380 | 0.447 | 0.522 | 0.570 | 0.662 |
| 21 | 0.292 | 0.370 | 0.436 | 0.509 | 0.556 | 0.647 |
| 22 | 0.284 | 0.361 | 0.425 | 0.497 | 0.544 | 0.633 |
| 23 | 0.278 | 0.353 | 0.416 | 0.486 | 0.532 | 0.621 |
| 24 | 0.271 | 0.344 | 0.407 | 0.476 | 0.521 | 0.609 |
| 25 | 0.265 | 0.337 | 0.398 | 0.466 | 0.511 | 0.597 |
| 26 | 0.259 | 0.331 | 0.390 | 0.457 | 0.501 | 0.586 |
| 27 | 0.255 | 0.324 | 0.383 | 0.449 | 0.492 | 0.576 |
| 28 | 0.250 | 0.318 | 0.375 | 0.441 | 0.483 | 0.567 |
| 29 | 0.245 | 0.312 | 0.368 | 0.433 | 0.475 | 0.558 |


Nominal Data
One Sample
Two Samples
Dependent Variable
Independent Variable Dependent Variable
Independent Groups
Dependent Groups
Chi-Square Goodness-of-fit
Chi-Square Test of Independence
Chi-Square Test of Independence
McNemar Test


Ordinal Data
One Sample
Multiple Samples
Independent Variable Dependent Variable
Independent Samples
Dependent Samples
Spearman's rs
Two Samples
Three + Samples
Two Samples
Three + Samples
Mann-Whitney U
Kruskal-Wallis H
Wilcoxon T
Friedman ANOVA by Ranks

- Check your data measurements
- Your data: do they meet necessary assumptions?
- Use non-parametric tests with caution
- Prioritize parametric tests before selecting non-parametric tests

THANK YOU